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DUAL SEMIGROUPS AND STRUCTURAL OPERATORS FOR PARTIAL FUNCTIONAL DIFFERENTIAL EQUATIONS WITH UNBOUNDED OPERATORS ACTING ON THE DELAYS

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1. Introduction. The purpose of this paper is to study a class of delay differential equations (DDE) of the form

$$\begin{aligned} \dot{x}(t) &= -Ax(t) + bA^{\mu}x(t-h) + \int_{-h}^{0} a(s)A^{\nu}x(t+s)\,ds, \quad \text{for } t > 0 \\ x(0) &= \phi^0, \quad x(s) = \phi^1(s) \quad \text{a.e. on } (-h,0), \end{aligned} \tag{1.1}$$

where $-A$ is the infinitesimal generator of an analytic semigroup on a Hilbert space X, A^{μ} and A^{ν}, $0 \le \mu,\ \nu < 1$ are fractional powers of A and b and $a(s)$ are scalar valued. Equations of this type were considered by several authors. Travis and Webb in [26], [27] treated (1.1) in the state space of continuous functions $C(-h,0;X^{\alpha})$, where $\alpha = \max(\mu,\nu)$ and X^{α} denotes the domain of A^{α} endowed with the graph norm. In this case $\phi^1 \in C(-h,0;X^{\alpha})$ and $\phi^0 = \phi^1(0)$. The case where all operators acting on the delayed part of the equation are bounded has been treated by several authors; see [25], for example. Kunisch and Schappacher in [KS] presented necessary conditions for DDE to generate a C_0-semigroup in the product space $X \times L^p(-h,0;X)$. The case where $\mu = \nu = 1$ is admitted in (1.1) has been considered in [1], [2], [6], [7], [8], for example. In [7], [8], DiBlasio, Kunisch and Sinestrari developed a state space theory in $Y \times L^2(-h,0;D(A))$, where Y is a real interpolation space between $D(A)$ and X and $D(A)$ denotes the domain of A endowed with the graph norm. Milota in [17] studied stability of DDE (1.1) with $0 \le \mu,\ \nu < 1$ and $h = \infty$. The first aim of this paper is to study wellposedness in a semigroup theoretic framework of DDE of the type (1.1) in the product space $X^{\beta} \times L^2(-h,0;X^{\alpha})$, where β is chosen in dependence of μ and ν.

Bernier, Delfour and Manitius in their study of DDE in $\mathbb{R}^n$ have shown that the so-called structural operator can be employed to describe concisely the influence of the delay part in (1.1) on the evolution of the trajectories. The structural operator

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