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ANALYTIC SOLUTIONS FOR A CLASS OF PDE'S WITH CONSTANT COEFFICIENTS

PAUL PEDERSEN

Department of Mathematics, University of Denver, Denver, Colorado 80208 USA

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Introduction. We demonstrate the existence of a linear associative algebra which can be used to find all analytic solutions to a given linear homogeneous PDE with constant coefficients.

It is well-known that every solution to Laplace's equation can be found as the real (or imaginary) part of functions of a complex variable. This result is remarkable in that our search for real-valued functions (harmonic functions) can be facilitated by considering analytic functions of a complex variable. In this paper we generalize this result to a large class of PDE's by exploring a concept of analytic function defined on a commutative algebra which is associated with the PDE. Analytic functions on this algebra will be associated with the PDE in the same way that analytic functions of a complex variable are associated with Laplace's equation. We then show how the analytic functions on the algebra can be used to generate all real analytic solutions to the associated partial differential equation.

We will consider real analytic solutions to the following partial differential equation in two variables:

$$\{b_0\frac{\partial^{m+1}}{\partial x^{m+1}}+\cdots+b_m\frac{\partial^{m+1}}{\partial x\partial y^m}+\frac{\partial^{m+1}}{\partial y^{m+1}}\}u(x,y)=0 \tag{1}$$

where the b_i are real. The reader will note that this PDE includes Laplace's equation (by letting $m=1$, $b_0=1$) and the wave equation (by letting $m=1$, $b_0=-1$) as special cases. The method discussed in this paper will allow one to explicitly find all polynomial solutions to this PDE. A unique characteristic of the method is that we need not factor the differential operator. It is relevant to note that Chapter 4 of Treves' book [1] discusses these polynomial solutions, but does not discuss how one might find them.

Throughout the following, whenever $b_0, b_1, \ldots, b_m$ are referred to they will mean these specific values for this specific differential equation. We will only consider the most interesting case which occurs when $b_0 \neq 0$.

A representation for the algebra. As noted above, the reader will recall that all harmonic functions can be found as the real (or imaginary) part of functions of

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