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ON A CLASS OF ABSTRACT DEGENERATE PARABOLIC EQUATIONS

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Abstract. We consider the abstract evolution equation $u'(t) = \varphi(t)Au(t) + f(t)$, $0 < t \leq T$, where $A : D(A) \subset X \to X$ generates an analytic semigroup and φ is a nonnegative continuous function. We obtain several existence and regularity results.

0. Introduction. The aim of this paper is studying existence, uniqueness and regularity of the solution of the initial value problem:

$$\begin{cases} u'(t) = \varphi(t)Au(t) + f(t), & 0 < t \leq T \\ u(0) = x \end{cases} \tag{0.1}$$

where $A : D(A) \subset X \to X$ generates an analytic semigroup e^{tA} in the Banach space X, $\varphi : [0,T] \to \mathbf{R}$ is a continuous nonnegative (possibly vanishing at $t = 0$) function, $x \in X$ and $f : [0,T] \to X$ is continuous. Such a problem has been considered by, among others, Friedman and Schuss in [5], in the case where X is a Hilbert space. They prove existence and uniqueness of a weak solution, assuming that φ is Holder continuous; they give also sufficient conditions in order that the solution belong to $H^1([\varepsilon, T]; X)$ for each $\varepsilon > 0$. On the contrary, we are mainly interested in classical and strict solutions, and we give several regularity results for u and u'. We consider also the problem of maximal regularity: in the cases where f is either Holder continuous with values in X, or bounded with values in a suitable interpolation space, we find that u' and $\varphi(\cdot)Au(\cdot)$ enjoy the same properties. Our approach is similar to the one developed in [7]; it is based on the explicit representation formula for the solution

$$u(t) = e^{A\int_0^t \varphi(r)\,dr}x + \int_0^t e^{A\int_s^t \varphi(r)\,dr} f(s)\,ds, \quad 0 \leq t \leq T. \tag{0.2}$$

In Section 1 we state assumptions and notation, and we study the properties of the function $w(t) = e^{A\int_0^t \varphi(r)\,dr}x$. Sections 2, 3 and 4 are devoted respectively to classical, strict and strong solutions to (0.1). In particular, in Section 3 we prove the

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