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DIFFERENTIAL INCLUSIONS WITH NON-CLOSED, NON-CONVEX RIGHT HAND SIDE

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Abstract. For a class of lower semicontinuous differential inclusions with non-closed, non-convex right hand side, the set of solutions is proved to be nonempty and connected. Existence of periodic solutions is also studied. Our results apply, in particular, to the problem $\dot{x} \in \operatorname{ext} F(x) \cap \operatorname{int} G(x)$, the right hand side being the intersection of the extreme and the interior points of two continuous multifunctions with compact, convex values.

1. Introduction.

Solutions of the differential inclusion

$$\dot{x}(t) \in F(t, x(t)) \tag{1.1}$$

can be obtained by constructing a selection $f(t,x) \in F(t,x)$ and solving the differential equation

$$\dot{x}(t) = f(t, x(t)). \tag{1.2}$$

When F is lower semicontinuous with compact but not necessarily convex values, this method was implemented in [2] by constructing a selection f which is directionally continuous; i.e., continuous with respect to the topology $\mathcal{T}^+$ generated by the half-open cones

$$\Gamma^M(t,x,\varepsilon) = \{(s,y);\ t \le s < t+\varepsilon,\quad \|x-y\| \le M(s-t)\}, \tag{1.3}$$

for some fixed $M > 0$. In the present paper, combining Baire's Category Theorem with the abstract selection theorem proved in [4], we obtain the existence of directionally continuous selections for maps of the form

$$F(t,x) = G(t,x) \setminus \bigcup_{k=1}^{\infty} R_k(t,x), \tag{1.4}$$

where G is lower semicontinuous with closed values, the graph of each multifunction R_k is closed and $R_k(t,x)$ is nowhere dense in $G(t,x)$, for each t, x. This technical refinement yields the existence of solutions for a new class of differential inclusions

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