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ON SOME NONLINEAR DISPERSIVE EQUATIONS IN SEVERAL SPACE VARIABLES

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1. Introduction. In this paper we treat nonlinear dispersive systems of the form

$$u_t - \Delta u_t = \operatorname{div} f(u), \qquad (x,t) \in \Omega \times (0,\infty), \tag{1.1}$$

subject to the Dirichlet boundary condition

$$u = 0, \quad (x,t) \in \partial\Omega \times (0,\infty), \tag{1.2}$$

and the initial condition

$$u(x,0) = u_0(x), \quad x \in \Omega, \tag{1.3}$$

where Ω is a bounded or unbounded smooth domain in $\mathbb{R}^n$, $n \geq 1$, and f is a nonlinear vector valued function $f(u) = (f_1(u), \ldots, f_n(u))$ of the class $C^2(\mathbb{R}, \mathbb{R}^n)$ satisfying the normalization condition $f(0) = 0$.

Benjamin, Bona, and Mahony [3] formulated the equation of the form

$$u_t + \Big(u + \frac{u^2}{2}\Big)_x - u_{xxt} = 0, \quad (x,t) \in \mathbb{R} \times (0,\infty) \tag{1.4}$$

which models long waves with small amplitude. This equation is understood to be a substitute model for the Korteweg-de Vries equation. The existence, uniqueness and regularity of the solutions of equation (1.4) were investigated under various conditions by a number of authors. For the related results we refer for instance to [7], [8] and their references. In fact, Oharu and Takahashi treated the problem (1.1)–(1.3) in the case of one space dimension in [7].

Equation (1.1) is not only a natural extension of the Benjamin-Bona-Mahony equation to the cases of higher space dimensions, but also it can be regarded as a pseudo-parabolic regularization of the single conservation law

$$u_t - \operatorname{div} f(u) = 0. \tag{1.5}$$

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