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ON THE SET OF SOLUTIONS TO LIPSCHITZIAN DIFFERENTIAL INCLUSIONS

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Introduction. In this paper we consider a Lipschitzian differential inclusion

$$x'(t) \in F(t,x), \quad x(t_0) = a, \tag{F}$$

where the values of F are compact but not convex. We prove that the map that associates to the initial point a the set of solutions to (F), $S_F(a)$, admits a selection, continuous from R^n to the space of absolutely continuous functions. The images of this map are sets that are not decomposable. In particular, the map from a to the attainable set at T, $A_T(a)$, admits a continuous selection. It is known that this map, in general, has no closed values.

Construction of the selection. We will use a further refinement of the selection technique of [1], [3], [6] and [8]. The main tools are a careful use of Liapunov's Theorem on the range of vector measures (see [6]) and of Filippov's extension of Gronwall's inequality ([5]; see also [2] p. 120). In what follows, $|a|$ is the Euclidean norm of a, $D(A,B)$ the Hausdorff distance of the sets A and B; $C(I)$ is the space of continuous mappings from I into R^n, with $\|f\|_C$ the sup norm. By $AC(I)$ we mean the space of absolutely continuous maps with the norm

$$\|f\|_{AC} = |f(t_0)| + \int |f'(s)|\, ds .$$

The map F will satisfy the following assumption.

Assumption (H). F is defined on an open Ω in R^{n+1}, bounded by M on it and such that

α) $t \to F(t,x)$ is measurable for fixed x;
β) $x \to F(t,x)$ is Lipschitzian with constant $K(t)$, $K \in L^1_{\text{loc}}$
γ) the value of F are compact;
δ) there exists a compact $A \subset R^n$ such that

$$\{(t,\, a + v(t-t_0)) : a \text{ in } A,\ v \text{ such that } |v| \le M,\ t \text{ in } [t_0,T]\} \subset \Omega .$$

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