Differential and Integral Equations, Volume 1, Number 4, October 1988, pp. 459-472.

STABILITY AND OSCILLATION OF NEUTRAL DELAY DIFFERENTIAL EQUATIONS WITH PIECEWISE CONSTANT ARGUMENT

E.C. PARTHENIADIS

Department of Mathematics, University of Rhode Island
Kingston, Rhode Island 02881-0816 USA

(Submitted by: Joseph Wiener)

Abstract. We established necessary and sufficient conditions for the asymptotic stability of the trivial solution and sufficient conditions for the oscillation of all solutions of the first order neutral delay differential equation with piecewise constant argument

$$\frac{d}{dt}\left(y(t)+py(t-1)\right)+qy([t-1])=0, \quad t\geq 0,$$

where p and q are real numbers and $[\cdot]$ designates the greatest-integer function.

We also obtained sufficient conditions for the oscillation of all solutions of the second order neutral delay differential equation with piecewise constant argument

$$\frac{d^2}{dt^2}\left(y(t)+py(t-1)\right)+qy([t-1])=0, \quad t\geq 0$$

and proved that the trivial solution is not asymptotically stable.

1. Introduction. In this paper we study the oscillatory behavior of solutions and the stability of the trivial solution of the first and second order neutral delay differential equations with piecewise constant argument (NEPCA)

$$\frac{d}{dt}\left(y(t)+py(t-1)\right)+qy([t-1])=0, \quad t\geq 0 \tag{1}$$

and

$$\frac{d^2}{dt^2}\left(y(t)+py(t-1)\right)+qy([t-1])=0, \quad t\geq 0, \tag{2}$$

where p and q are real numbers and $[\cdot]$ designates the greatest-integer function.

First order equations with piecewise constant arguments, of non-neutral type have been the subject of many recent investigations originated by Cooke and Wiener [2] and Shah and Wiener [4]. See also [1] and the references cited therein. These equations may have applications in vertically transmitted diseases. They are also connected to difference equations as

Received November 24, 1987.
AMS(MOS) Subject Classifications: Primary 34K15, Secondary 34C10.