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EVOLUTION OPERATORS AND STRONG SOLUTIONS OF ABSTRACT LINEAR PARABOLIC EQUATIONS

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Abstract. We consider the linear non-autonomous Cauchy problem of parabolic type in a Banach space E, under general assumptions which allow the domains of the operators to be non-constant in t and not dense in E. We study the regularity properties of the evolution operator, and prove existence, uniqueness and sharp regularity results for strong solutions. Applications to parabolic partial differential equations are also given.

0. Introduction. Let E be a Banach space. We are concerned with the linear parabolic non-autonomous Cauchy problem

$$\begin{cases} u'(t) - A(t)u(t) = f(t), & t \in [s,T] \\ u(s) = x. \end{cases} \tag{0.1}$$

Here, $T > 0$, $s \in [0,T]$ and $x \in E$, $f : [s,T] \to E$ are prescribed data, whereas $\{A(t)\}$ is a family of closed linear operators in E, which are generators of analytic semigroups and whose domains $D_{A(t)}$ may change with t and be not dense in E. In [3], we studied existence, uniqueness, and maximal regularity of strict and classical (i.e., continuously differentiable) solutions of (0.1), and in [4] we constructed the evolution operator $U(t,s)$ for problem (0.1). In both cases, the initial point was $s = 0$, but the general situation $s \in [0,T]$ requires no substantial changes. Here, under the same assumptions of those papers, we consider the variation of parameters formula

$$u(t) := U(t,s)x + \int_s^t U(t,r)f(r)\,dr, \quad t \in [s,T], \tag{0.2}$$

and show that u is the unique strong solution (see Definition 1.6 (c) below) of (0.1), if and only if $x \in \overline{D_{A(s)}}$ and $f \in C([s,T],E)$. Furthermore, we prove very precise regularity results, both in time and in space, for the function (0.2); such results generalize those of [7], [14], and [2], and are sharper than the similar ones obtained in [1] under hypotheses which are independent of ours (see Remark 4.5 below). In addition, we shortly consider classical solutions, showing that formula (0.2) holds for them too, under very general conditions on the data (compare with [4, Remark 2.3]). We note that, as in our previous papers, [3]

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