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TANGENT DIRECTIONS FOR A CLASS OF NONLINEAR EVOLUTION EQUATIONS

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Abstract. A class of nonlinear evolution equations involving locally Lipschitzian term is discussed. By using a global linearization iterative method, the existence of tangent directions to the set determined by nonlinear evolution equations is proved in two cases: without and with the control function involved. This result is then applied to the problem of optimal control with nonlinear evolution equation as constraint.

Introduction. The problem of tangent directions to the sets determined by some equation is a very important problem in optimization and optimal control. In the famous results obtained by Dubovitskii, Milyutin, Girsanov (cf. [6]), Joffe, Tikhominov (cf. [7]) and others, considering only one equality constraint in the form of ordinary differential equation, the proofs of the existence of tangent directions are based on the Lusternik theorem. The Lusternik theorem is also applied to optimization and optimal control problems with more than one equality constraint as in [8], [9], [10], [11], [12].

In [1], some generalization of the Lusternik theorem is proved by using the method of contractor directions under essentially weaker assumptions about differentiability than the Lusternik theorem requires. This generalization is applied to the problems of optimization and optimal control in [1], [13], [14] and [15].

However, none of these results (i.e., Lusternik theorem and its generalization) are applicable to the problems of optimal control with nonlinear evolution equation as constraint because of too strong assumptions that these results require.

In [3] and [5] the global linearization iterative method from [2] is applied to prove the existence of tangent directions for nonlinear evolution equation and quasilinear evolution equation, respectively. An application to the optimal control problem of quasilinear evolution equation as constraint is also considered.

In this paper, the results from [3] are extended to a more general class of nonlinear evolution equation with locally Lipschitzian term and the existence of tangent directions to the set determined by this equation is proved. Next, as an application, the local extremum principle for the optimal control problem with these general nonlinear evolution equation as constraint is proved.

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