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PERIODIC BOUNDARY VALUE PROBLEM FOR SOME DUFFING EQUATIONS

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Abstract. We consider the periodic problem for the forced Duffing equation

$$u'' + cu' + g(t,u) = 0,$$

$$u(0) = u(2\pi), \quad u'(0) = u'(2\pi)$$

where g is 2π-periodic in t. We study existence of solution under the hypothesis that $c \neq 0$ and

$$\limsup_{|u|\to\infty} \left| \frac{g(t,u)}{u} \right| \leq 1 + c^2 .$$

We also consider conditions under which the set of solutions is an R_δ .

1. Some existence results. In recent years much work has been done concerning the existence of periodic solutions to the Duffing equation

$$\begin{aligned} &u'' + cu' + g(t,u) = 0, \\ &u(0) = u(2\pi), \quad u'(0) = u'(2\pi) \end{aligned} \tag{1}$$

(cf. [2–11, 13, 14, 16]).

In this section we study a new existence result, assuming that $g(t,u)$ is a Carathéodory function (i.e., continuous in u and measurable in t), 2π-periodic in t, which in addition satisfies the following conditions.

(g_1) For each $R > 0$ there exists a function $a \in L^2(0,2\pi)$ such that for all $(t,u) \in [0,2\pi] \times [-R,R]$,

$$|g(t,u)| \leq a(t).$$

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