Differential and Integral Equations, Volume 1, Number 4, October 1988, pp. 391-398.

CONJUGATE POINTS OF FOURTH ORDER DIFFERENTIAL EQUATIONS WITH IMPULSES

KURT KREITH
Department of Mathematics, University of California
Davis, California 95616 USA

(Submitted by : F.V. Atkinson)

Abstract. Selfadjoint fourth order differential equations of the form

$$(p_2(t)y'')'' - (p_1(t)y')' + p_0(t)y = 0$$

are considered in a generalized context which allows for distributional terms in the coefficients. Criteria are established for the existence and non-existence of conjugate points for such equations.

1. Introduction. The classical existence theory for differential equations has, in a variety of contexts, been extended to allow for solutions which satisfy the underlying equation in only a generalized sense. Such theories of "generalized differential equations" can be developed (more or less equivalently) in terms of novel theories of integration [5], the introduction of measures [7] or via the theory of distributions [1]. (See also the very useful survey [9]).

While there is a substantial literature dealing with existence, uniqueness and stability for generalized differential equations, qualitative theories of oscillation, conjugacy and asymptotic behavior have tended to be pursued in a more classical context. Thus, in the case of selfadjoint fourth order equations of the form

$$(p_2(t)y'')'' - (p_1(t)y')' + p_0(t)y = 0 \tag{1.1}$$

the seminal paper of Leighton and Nehari [6] requires that p_2, p_1 and p_0 be of class C'', C' and C, respectively, on the interval under consideration. Most subsequent studies of such equations have involved similar limitations.

In the study of conjugate points, there are, of course, strong incentives for assuming sufficient regularity to enable one to bring powerful variational tools to bear. For second order Sturm-Liouville equations of the form

$$-(p_1(t)y')' + p_0(t)y = 0, \tag{1.2}$$

this means that $p_0(t)$ may include distributional terms corresponding to delta functions (leading to discontinuities in $y'(t)$) but not derivatives of delta functions (which lead to

Received September 17, 1987.
AMS(MOS) Subject Classifications: 34C10.