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A NONLINEAR SYSTEM FOR PHASE CHANGE WITH DISSIPATION

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Abstract. A nonlinear system modeling a phase change problem with dissipation is investigated. The model is derived with the thermodynamical theory of continuum mechanics; it includes among other features superheating and supercooling effects or irreversible phase changes. One of the equations is doubly nonlinear, with a nonlinearity on the time derivative of one of the unknowns and the resulting system is non-monotone in $(L^2)^2$ or in $(H^{-1})^2$. Uniqueness of the solution is proved using the accretivity of the system in $(L^1)^2$. The existence of a solution is shown through a regularization of one the nonlinearities. In this case, such a method permits to weaken the customary assumption of the L^1-framework.

1. Introduction In this paper, existence and uniqueness of the solution of a nonlinear system modelling some dissipative phase change phenomena is established. The system investigated is the following:

$$c(x)\frac{d\theta}{dt} + \frac{d\chi}{dt} - \operatorname{div}(k(x)\nabla\theta) = 0 \quad \text{in } \Omega \times (0,T) \tag{1.1}$$

$$\frac{d\chi}{dt} + \partial\phi(x, \frac{d\chi}{dt}) + \partial\psi(x,\chi) \ni \theta \quad \text{in } \Omega \times (0,T) \tag{1.2}$$

$$\theta = 0 \quad \text{on } \partial\Omega \times (0,T),$$

$$\theta(t=0) = \theta_0, \quad \chi(t=0) = \chi_0 \quad \text{in } \Omega,$$

where $\partial\phi(x,\cdot)$ and $\partial\psi(x,\cdot)$ are the subdifferentials of two convex functions $\psi(x,\cdot)$ and $\phi(x,\cdot)$ on $\mathbb{R}$.

Section 2 is devoted to the introduction of the physical problem under investigation. Also the usual techniques of thermodynamics are used to derive the model (1.1) and (1.2) in this section. Thermodynamical considerations allow one to make precise some properties

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