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AN ESTIMATE FOR THE MINIMA OF THE FUNCTIONALS OF THE CALCULUS OF VARIATIONS

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1. We consider a functional of the Calculus of Variations of the following form

$$J(w) = \int_G [f(x, Dw(x)) - g(x, w(x))]\, dx \tag{1.1}$$

where

i) G is a bounded open subset of R^n,

ii) $f : (x, z) \in G \times R^n \to R$ is a Carathéodory function,

iii) $g : (x, s) \in G \times R \to R$ is measurable in x and differentiable in s.

Let $A : [0, +\infty) \to R$ be a convex function such that $\lim_{r \to 0}(A(r)/r) = 0$ and consider a function $u \geq 0$ which minimizes (1.1) in the Orlicz space $W_0^{1,A}(G)$ (this definition is given in Section 2). Here, we prove an a priori estimate for u. Therefore, we are not concerned with the existence problem of minima, but assuming the existence of a minimum of (1.1), we seek a priori bounds for it.

Before we state our result more precisely, we recall that for each function $\phi \in L^1(G)$ its Schwarz symmetrized, denoted by ϕ^*, is defined in the ball G^* centered at the origin and with the same measure as G. In Section 2, we give some definitions and preliminaries. In Section 3, we prove the following theorem.

Theorem. *Let i), ii) and iii) hold. Moreover, assume that*

iv) there exists a function $A(r)$ with the above properties such that

$$\liminf_{\epsilon \to 0^-} \frac{f(x, (1+\epsilon)z) - f(x, z)}{\epsilon} \geq A(|z|) \quad \forall x, z.$$

v) The partial derivative $g_s(x, s)$ of $g(x, s)$ with respect to s satisfies

$$g_s(x, s) \leq g_s(x, 0) \quad \forall s \geq 0, \quad \text{for a.e. } x \in G$$

with $g_s(x, 0) \in L^1(G)$. Then, if $u \geq 0$ is a minimum in $W_0^{1,A}(G)$ for the functional (1.1),

$$u^*(x) \leq v(x) \quad \text{for a.e. } x \in G^*,$$

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