Differential and Integral Equations, Volume 2, Number 3, July 1989, pp. 310-325.

ON SPATIAL ENERGY DECAY FOR QUASILINEAR BOUNDARY VALUE PROBLEMS IN CONE-LIKE AND EXTERIOR DOMAINS

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(Submitted by: A.R. Aftabizadeh)

Abstract. A number of boundary value problems for second order quasilinear partial differential equations in divergence form, in cone-like domains and exterior domains, in two and three dimensions, are considered. For the cone-like domains, homogeneous data of either the Dirichlet, Neumann, or mixed type are prescribed on the lateral sides. New results are obtained concerning the spatial decay of the energy (Dirichlet norm of the solution) at infinity and its maximum rate of growth near the finite end of the domain.

1. Introduction. During the last 25 years, much research has been done on spatial decay behavior of solutions of elliptic boundary value problems in semi-infinite domains. A comprehensive review of the work up to the early eighties is given by Horgan and Knowles [1]. Among the subsequent papers on the subject are [2]-[7] and also some of the references therein.

The equations considered here, as in [5]-[7], are single second order equations in divergence form:

$$\frac{\partial}{\partial x_1}\left[\rho(\mathbf{x},u,\nabla u)\frac{\partial u}{\partial x_1}\right]+\frac{\partial}{\partial x_2}\left[\rho(\mathbf{x},u,\nabla u)\frac{\partial u}{\partial x_2}\right]+\frac{\partial}{\partial x_3}\left[\rho(\mathbf{x},u,\nabla u)\frac{\partial u}{\partial x_3}\right]=0\ , \tag{1.1}$$

for a wide range of positive functions ρ. The conditions on ρ are general enough so that (1.1) need not necessarily be elliptic as long as a solution exists which satisfies the given conditions of the problem. The boundary data are of either the Dirichlet, Neumann, or mixed type.

Many important equations of mathematical physics are included in (1.1). For example, when $\rho = 1$, one obtains Laplace's equation, when $\rho = \rho(\mathbf{x}, u)$, (1.1) corresponds to the nonlinear steady-state diffusion equation, and when $\rho = [1 + |\nabla u|^2]^{-\frac{1}{2}}$, the minimal surface equation is obtained.

Throughout this paper, standard index notation will be used, and the usual summation convention will be followed. Thus, we write

$$u,_j = \frac{\partial u}{\partial x_j}\ ;$$

$$[\rho u,_\beta],_\beta = [\rho u,_1],_1 + [\rho u,_2],_2\ ;$$

$$[\rho u,_j],_j = [\rho u,_1],_1 + [\rho u,_2],_2 + [\rho u,_3],_3\ ;$$

Received June 6, 1988, revised November 16, 1988.
AMS Subject Classifications: 35B40, 35B05, 35J25, 35J60.