OSCILLATION OF DISCRETE ANALOGUES OF DELAY EQUATIONS*

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Abstract. Oscillation and nonoscillation criteria are established for first order linear and nonlinear difference equations with delay. Some of the conditions are shown to be sharp. Applications are also given to certain nonhomogeneous equations.

1. Introduction. In a number of recent papers [1, 6-9, 11, 12], the oscillatory and asymptotic behaviour of solutions of certain linear and nonlinear difference equations has been extensively investigated. It turns out that many (but not all, see [7]) of the substantial criteria for differential equations have discrete analogues. Further, criteria have also been obtained for the oscillatory and nonoscillatory behaviour of differential delay equations which may also be considered as analogues of criteria in the ordinary differential equations case [10]. It is the purpose of this paper to combine these two approaches in order to obtain oscillation criteria for discrete analogues of first order delay differential equations.

We consider in section 2 first order difference equations of the form

$$y_{n+1} - y_n + p_n y_{n-m} = 0 \tag{1.1}$$

and

$$y_{n+1} - y_n + p_n f(y_{n-m}) = 0 \tag{1.2}$$

where $n = 1, 2, \ldots$, and m is a positive integer. Equation (1.2) has also been considered in the numerical analysis of certain functional differential equations ([2]). Equations (1.1) and (1.2) are discrete analogues of

$$y'(t) + p(t)y(t - m) = 0 \tag{1.3}$$

and

$$y'(t) + p(t)f(y(t - m)) = 0 \tag{1.4}$$