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HEREDITARY CONTROL SYSTEMS GOVERNED BY INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. This paper examines nonlinear, hereditary control systems, with dynamics described by an integrodifferential equation. It is proved that the set of trajectories is compact in an appropriate space of continuous functions and under some additional hypotheses on the velocity field, that it is also connected. Analogous properties are obtained for the attainable sets, and it is also shown that the attainability multifunctions is Hausdorff continuous in the time variable. Then a relaxation and a "bang-bang" type theorem is proved. Finally some optimal control problems are solved.

1. Introduction. The purpose of this paper is to examine hereditary control systems (i.e., control systems with memory), governed by an integrodifferential equation. So the state velocity does not only depend on the past history of the state, but also on the past values of the control function. Our work extends earlier works of Angell [1], Cesari [5], Hermes-LaSalle [8] and Oguztoreli [13].

The hereditary control system under consideration is the following:

$$\begin{cases} \dot{x}(t) = g(t, x_t) + \displaystyle\int_0^t K(t,s) f(s, x(s), u(s))\, ds, \quad \text{a.e.. on } \; T = [0, b] \\ x(v) = \phi(v) \quad \text{for} \quad v \in T_0 = [-r, 0], \quad u(t) \in U(t, x(t)) \quad \text{a.e.} \end{cases} \tag{*}$$

Here, $x : T_b = [-r, b] \to X$ is the state function and $x_t(\cdot) = x|_{[t-r,t]}(\cdot)$, i.e., represents the past history of the state up to time t.

In the next section we will recall some basic defintions from the theory of multifunctions that we will need in the sequel and we will state the main hypotheses concerning the data of our system (*). In section 3, we study in detail the structure of the set of trajectories of (*) and of the corresponding attainable set. In section 4, we prove a relaxation result which, roughly speaking, says that convexification of the velocity field does not alter the reachability properties of the system. Also, we prove a "bang-bang" type theorem. Finally, in section 5, we solve various optimization problems involving system (*).

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