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ON THE TWO DIMENSIONAL SINGULAR INTEGRAL EQUATIONS

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Abstract. Applying the "equivalent curve" method provided by the author, this paper discusses general solutions for the two dimensional singular integral equation

$$\omega(z) - \frac{q(z)}{\pi} \int_G \int \frac{\bar{\omega}(\xi)}{(\bar{\xi} - \bar{z})^2}\, d\sigma_\xi = f(z),$$

here G is a simply connected region in the complex plane.

Suppose G is a simply connected region in the complex plane E, its boundary Γ is formed by finite arcs in C^1_α. In this paper we discuss general solutions for the following two dimensional singular integral equation on $\bar{G}$:

$$\omega(z) - \frac{q(z)}{\pi} \int_G \int \frac{\bar{\omega}(\xi)}{(\bar{\xi} - \bar{z})^2}\, d\sigma_\xi = f(z). \tag{1}$$

Here the solution $\omega(z)$ is in the space

$$V(\bar{G}) = \{\omega(z) : \omega \in L_1(\bar{G}) \cap L_p(\bar{G}/\{a_1, \cdots, a_Q\}),\ p > 2\},$$

and $q(z)$ satisfies conditions

(i) $|q(z)| = 1$, $z \in \bar{G}/\{a_1, \cdots, a_Q\}$;

(ii) in $\bar{G}/\{a_1, \cdots, a_Q\}$, $q(z)$ has continuous partial derivatives which may have poles with degree smaller than 2 at $a_1, \cdots, a_Q$;

(iii) $f(z) \in V(\bar{G})$.

The equation (1) is solved by N.H. Vekua [1] under the condition $|q(z)| < 1$ and is discussed by A. Džuraev [2] and N.N. Komjak [3], [4] in the case $|q(z)| > 1$ and $q(z) = e^{i\lambda}$ (here λ is a real constant).

Denote

$$(T\omega)(z) = -\frac{1}{\pi} \int_G \int \frac{\omega(\xi)}{\xi - z}\, d\sigma_\xi \quad \text{for } \omega \in V(G),$$

then $(T\omega)(z)$ has generalized derivatives [1]

$$\frac{\partial (T\omega)(z)}{\partial \bar{z}} = \omega(z), \quad \frac{\partial (T\omega)(z)}{\partial z} = -\frac{1}{\pi} \int_G \int \frac{\omega(\xi)}{(\xi - z)^2}\, d\sigma_\xi.$$

It is easy to prove that the function $(T\omega)(z)$ is analytic in $E/\bar{G}$, continuous in $E/\{a_1, \cdots, a_Q\}$ and $(T\omega)(\infty) = 0$.

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