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SCHAUDER ESTIMATES AND EXISTENCE THEORY FOR ENTIRE SOLUTIONS OF LINEAR PARABOLIC EQUATIONS

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Abstract. Solutions in $\mathbb{R}^{n+1}$ of the linear, parabolic, nonhomogeneous partial differential equation

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{i,j}\frac{\partial^2\phi}{\partial x_i\partial x_j}+\sum_{i=1}^{n} b_i\frac{\partial\phi}{\partial x_i}+c\phi-\frac{\partial\phi}{\partial t}=f,$$

and the related homogeneous equation are investigated. By entire solutions is meant solutions of these equations defined in all of $\mathbb{R}^{n+1}$. Schauder-type a priori estimates are developed for entire solutions with prescribed behavior at infinity. These estimates lead to an existence theory for entire solutions with certain behavior required at infinity. Uniqueness of these solutions follows from the maximum principle.

1. Introduction. We investigate here solutions in $\mathbb{R}^{n+1}$ of the linear, parabolic, nonhomogenous, variable coefficient, partial differential equation

$$\begin{aligned}\mathcal{L}\phi :&= a\cdot\mathcal{D}^2\phi+b\cdot\mathcal{D}\phi+c\phi-\partial\phi/\partial t\\ &=\sum_{i=1}^{n}\sum_{j=1}^{n} a_{i,j}\frac{\partial^2\phi}{\partial x_i\partial x_j}+\sum_{i=1}^{n} b_i\frac{\partial\phi}{\partial x_i}+c\phi-\frac{\partial\phi}{\partial t}=f\end{aligned}\tag{NH}$$

and the related homogeneous equation,

$$\mathcal{L}\phi=0.\tag{H}$$

Following the prevailing terminology in the literature, we refer to solutions of these equations defined in all of $\mathbb{R}^{n+1}$ as entire solutions. We shall develop Schauder-type a priori estimates for the entire solutions with prescribed behavior at infinity. These estimates, of perhaps independent interest themselves, lead to an existence theory for entire solutions with certain behavior required at infinity. Uniqueness of these solutions follows from the maximum principle.

The coefficients a, b, c of $\mathcal{L}$ are assumed to be Hölder-continuous in $\mathbb{R}^{n+1}\cup\{\infty\}$: and as $x\to\infty$, the matrix a approaches the $n\times n$ identity matrix I, the vector b approaches the $1\times n$ zero vector and the scalar c approaches zero. Thus $\mathcal{L}$ approaches the heat operator

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