Differential and Integral Equations, Volume 2, Number 1, January 1989, pp. 111-121.

SMOOTH SOLUTIONS FOR AN INTEGRO-DIFFERENTIAL EQUATION OF PARABOLIC TYPE

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(Submitted by: G. Da Prato)

Abstract. In this paper we shall discuss the solvability of general linear integro-differential equation of parabolic type in the Hölder class $H^{2+\alpha,1+\alpha/2}(\bar{Q}_T)$ by employing the basic classical theory of parabolic equations. The questions of existence, uniqueness and stability are studied. We also apply our results to some mixed type boundary value problems arising in the study of the propagation of sound in viscous media.

1. Introduction. In the past decades many papers have appeared in the journals [1, 2, 3, 8, 9, 17, 19, 20] (and the references cited there) concerning the following type of integro-differential equation

$$u_t(x,t) = A(x,t)u(x,t) + \int_0^t B(x,t,\tau)u(x,\tau)\,d\tau + f(x,t), \quad \text{in } Q_T \tag{1.1}$$

supplemented with initial-boundary value conditions. Here $Q_T = \Omega \times (0,T]$, $T > 0$ and Ω is a bounded open subset of R^n with regular boundary $\partial\Omega$, $A(x,t)$ and $B(x,t,\tau)$ are two linear differential operators of second order. Equation (1.1) can also be written in the form

$$\begin{aligned} u'(t) &= A(t)u(t) + \int_0^t B(t,\tau)u(\tau)\,d\tau + f(t), \quad 0 < t \leq T, \\ u(0) &= u_0 \end{aligned} \tag{1.2}$$

in a Banach space E. The setting (1.2), together with vanishing boundary conditions, allowed the authors of [1, 2, 9, 20] to employ abstract semi-group methods to show that under some appropriate assumptions on the data there exists a unique solution u in the Banach space E. The reader can also find the discussion of non-linear counterparts of (1.1) in [8, 14] and the references given there.

In this paper we shall take a different approach by treating (1.1) as a parabolic equation with the integral term as a perturbation and employing the classical Picard iteration. We

Received March 31, 1988.
AMS Subject Classifications: 35K, 35R.