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ON FOURTH ORDER BOUNDARY VALUE PROBLEMS ARISING IN BEAM ANALYSIS

RAVI P. AGARWAL

Department of Mathematics, National University of Singapore, Kent Ridge, Singapore 0511

(Submitted by: C. Corduneanu)

Abstract. We consider a general fourth order nonlinear ordinary differential equation together with two point boundary conditions which occur in the deflection of a beam rigidly fastened at left and simply fastened at right. For this general boundary value problem, we provide necessary and sufficient conditions for the existence and uniqueness of the solutions. We also obtain upper estimates on the length of interval so that Newton's method converges quadratically to the unique solutions. Some of our results are the best possible in their frame.

1. Introduction. Imposing some ideal conditions, the deflection of a beam rigidly fastened at left and simply fastened at right leads to a fourth order differential equation (in simplest form) [4, 7-9, 11, 19, 22]

$$x'''' = \alpha^4 x \quad (\alpha > 0) \tag{1.1}$$

together with the two point boundary conditions

$$x(a) = A,\ \ x'(a) = B,\ \ x(b) = C,\ \ x''(b) = D. \tag{1.2}$$

The problem (1.1), (1.2) has a unique solution if and only if

$$\cosh\alpha(b-a)\sin\alpha(b-a) - \cos\alpha(b-a)\sinh\alpha(b-a) = p \neq 0 \tag{1.3}$$

and can be expressed in terms of elementary functions

$$\begin{aligned} x(t) = A\cosh\alpha(t-a) + \frac{B}{\alpha}\sinh\alpha(t-a) &+ \beta\,(\cos\alpha(t-a) - \cosh\alpha(t-a)) \\ &+ \gamma\,(\sin\alpha(t-a) - \sinh\alpha(t-a)) \end{aligned} \tag{1.4}$$

where the constants β and γ appear as

$$\begin{aligned} \beta = \frac{1}{2p}\Big[\Big(2A\cosh\alpha(b-a) + \frac{2B}{\alpha}\sinh\alpha(b-a) - \frac{D}{\alpha^2} - C\Big)\sin\alpha(b-a) \\ + (\frac{D}{\alpha^2} - C)\sinh\alpha(b-a)\Big], \end{aligned}$$

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