Differential and Integral Equations, Volume 2, Number 1, January 1989, pp. 57-62.

COMPACT EVOLUTION OPERATORS

N.H. PAVEL
Department of Mathematics, Ohio University, Athens, Ohio 45701 USA

(Submitted by G. Webb)

In this paper, we give a characterization of compactness of the Evolution Operator $U(t,s)$ generated by a family of nonlinear (possibly multivalued) operators $\{A(t), 0 \leq t \leq T\}$ of dissipative type. This is an extension of a result of Brezis [2] on the compactness of the semigroup $S_A(t)$ generated by an m-dissipative operator A via the exponential formula of Crandall-Liggett [3].

1.Preliminaries. Statements of the results. Let X be a real Banach space of norm $\| \ \|$. Recall some notations and definitions (for details we refer to [6], [7]).

$$\langle y,x\rangle_{\bar{s}} = \lim_{h\downarrow 0} \frac{\|x+hy\|^2 - \|x\|^2}{2h}, \quad \langle y,x\rangle_{+} = \lim_{h\downarrow 0} \frac{\|x+hy\| - \|x\|}{h}, \tag{1.1}$$

$$\langle y,x\rangle_{i} = \lim_{h\downarrow 0} \frac{\|x+hy\|^2 - \|x\|^2}{2h}, \quad \langle y,x\rangle_{-} = \lim_{h\downarrow 0} \frac{\|x+hy\| - \|x\|}{h}, \tag{1.2}$$

where x and y are elements of X. Clearly,

$$\langle y,x\rangle_{\bar{s}} = \|x\|\langle y,x\rangle_{+}\,;\, \langle y,x\rangle_{i} = \|x\|\langle y,x\rangle_{-}\,,\, \langle y,x\rangle_{+} = \frac{\|x+hy\| - \|x\|}{h}, \quad h > 0.$$

It is known that

$$\langle y,x\rangle_{\bar{s}} = \sup\{x^*(y)\,;\, x^* \in J(x)\}, \quad \langle y,x\rangle_{i} = \inf\{x^*(y)\,;\, x^* \in J(x)\}, \tag{1.3}$$

where J is the duality mapping of X.

Now, let $\{A(t)\,;\, 0 \leq t \leq T\}$ be a family of nonlinear (possibly multivalued) operators $A(t) : D(A(t)) \subset X \to 2^X$ satisfying the hypotheses

(C1) $R(I - hA(t)) = X$, for all $h > 0$ and $t \in [0,T]$

(C2) There are two continuous functions $f : [0,T] \to X$ and $L : R_+ \to R_+$ such that

$$\langle y_1 - y_2\,,\, x_1 - x_2\rangle_i \leq \|f(t) - f(s)\|\, \|x_1 - x_2\| L(\max\{\|x_1\|\,,\, \|x_2\|\})$$

for all $0 \leq s \leq t \leq T$, $[x_1, y_1] \in A(t)$ and $[x_2, y_2] \in A(s)$

(C3) If $t_n \uparrow t$, $x_n \in D(A(t_n))$ and $x_n \to x$, then $x \in \overline{D(A(t))}$.

Received Feburary 24, 1988.
AMS Subject Classifications: 47H06, 47H99.