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PERIODIC BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF FIRST ORDER IMPULSIVE DIFFERENTIAL EQUATIONS

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1. Introduction. The theory of impulsive differential equations is emerging as an important area of investigation since it is a lot richer than the corresponding theory of differential equations [6]. Furthermore, such equations appear to represent a natural framework for mathematical modelling of several real world phenomena. For example, the reproduction of the microorganisms suggests that periodic solutions exist under certain impulsive conditions. In spite of its importance, the development of the theory has been slow due to special features possessed by impulsive differential systems in general, such as pulse phenomena, confluence and loss of autonomy [6].

One of the traditional branches of investigation in the qualitative theory of differential equations is the study of periodic solutions. Because of the peculiarities mentioned above, the investigation of periodic solutions of impulsive differential systems has been restricted to the use of successive approximations [3, 7, 8, 9].

We discuss in this paper the periodic boundary value problem for a system of impulsive differential equations in which impulses occur at fixed times and prove, with the help of upper and lower solutions, that the problem has a solution lying between the upper and lower solutions. We also consider the theory of impulsive differential inequalities relative to periodic boundary value problems. We note that the method of upper and lower solutions has been utilized to study various initial and boundary value problems for differential equations without impulses. However, these discussions depend on abstract methods [1, 4, 5]. Our proofs employ simple new ideas and avoid abstract arguments. Our approach here and in [2] is entirely based on calculus methods and it is therefore easy to understand the basic ideas involved, and at the same time, acheive the same goals as that of abstract methods. Moreover, our methods provide new proofs even to the same problems for systems of differential equations.

Before closing this section, we give the following simple example to show that solutions of impulsive differential equations do exhibit peculiar specific features.

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