Differential and Integral Equations, Volume 2, Number 1, January 1989, pp. 13-20.

STABILITY IN TERMS OF TWO MEASURES FOR FUNCTIONAL DIFFERENTIAL EQUATIONS

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(Submitted by: V. Lakshmikantham)

1. Introduction. In the investigation of stability results for functional differential equations using Lyapunov functionals, one usually employs the two norms, $|\phi(0)|$ and $\|\phi\| = \max_{-\tau\le s\le 0}|\phi(s)|$, where $\phi \in C[[-\tau,0],R^n]$ to impose conditions on Lyapunov functional. Examining several standard examples known, Burton and Hatvani [1,2] recently discovered that a more flexible approach results when one uses the L^2 norm instead of $\|\phi\|$ and proved some interesting results by this approach. In [4], stability results for ordinary differential equations are presented in terms of two measures employing perturbing families of Lyapunov functions and incorporating the ideas involved in recent advances in stability theory relating to Marachkov's and Movchan's theorems. In this paper, we consider stability properties in terms of two measures and Lyapunov functionals extending Marachkov's theorem in the light of Salvadori's [5] generalization to functional differential systems.

2. Preliminaries. Let us list the following notations and definitions for convenience.

$$K = [\sigma \in C[R_+,R_+],\ \sigma(u) \text{ is strictly increasing and } \sigma(0)=0]\ .$$

$$CK = \left[\sigma \in C[R_+^2,R_+],\ \sigma(t,u)\in K \text{ for each } t\in R_+\right]\ .$$

$$\Gamma = \left[h \in C[R_+\times R^n,R_+],\ \inf_{(t,x)\in R_+\times R^n} h(t,x)=0\right]\ .$$

$$\Gamma_0 = \left[h_0 \in C[R_+\times C,R_+],\ \inf_{\phi\in C} h_0(t,\phi)=0,\ \text{where } C=[\phi\in C[[-\tau,0],R^n]\right]\ .$$

Consider the following functional differential system

$$x'(t) = F(t,x_t)\,, \tag{2.1}$$

where $F\in C[R_+\times C,R^n]$, $x_t = x(t,s)$ for $s\in[-\tau,0]$ and τ is a fixed positive constant.

Definition 2.1. *For any* $V \in C[R_+\times C,R_+]$, *define*

$$\dot V(t,\phi) = \lim_{h\to 0^+}\sup\frac{1}{h}\left[V(t+h,x_{t+h}(t,\phi)) - V(t,\phi)\right]\ .$$

where it is understood that $x(t,\phi)$ *is any solution of (2.1) with an initial function* ϕ *at time* t.

Received January 29, 1988.
AMS Subject Classifications: 34K20.