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ON A THIRD-ORDER THREE-POINT BOUNDARY VALUE PROBLEM AT RESONANCE

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Abstract. D. Krajcinovic modelled the static deflection of a three-layered elastic beam by a linear third order, three-point, boundary value problem involving only the first order and third order derivatives of the displacement. This paper is concerned with the existence and uniqueness of solutions of some third order, three-point, nonlinear boundary value problems which are at resonance, in the sense that the associated linear boundary value problem has non-trivial solutions. The methods used involve second-order integro-differential boundary value problems and a use of the Leray-Schauder continuation theorem.

1. Introduction. This paper is motivated by the model equation describing the deflection of a three-layer beam formed by parallel layers of different materials given by D. Krajcinovic in [8]. According to Krajcinovic, the state of static deflection of an equally-loaded three-layered beam is described by

$$\begin{cases} u''' - k(K_2 A_e - K_1^2)u' + a = 0, \\ u'(0) = u'(1) = u(\frac{1}{2}) = 0, \end{cases} \tag{1.1}$$

where K_1, K_2 are shear parameters, A_e is the cross-sectional area of the beam and k, a are other physical parameters related to the elasticity of the layers. Krajcinovic assumed that $K_2 A_e - K_1^2 > 0$. It is clear from (1.1) that the assumption of $K_2 A_e - K_1^2 > 0$ is rather restrictive, and interesting physical situations arise when $K_2 A_e - K_1^2$ is in fact negative. To this end, the author with A.R. Aftabizadeh and J.M. Xu [2] recently studied the following nonlinear third-order three-point boundary value problem:

$$\begin{cases} u''' + f(u')u'' = g(x, u, u', u'') + e(x), \\ u'(0) = u'(1) = u(\eta) = 0, \quad 0 \le \eta \le 1, \end{cases} \tag{1.2}$$

where $f : \mathbf{R} \to \mathbf{R}$ is continuous, $g : [0,1] \times \mathbf{R}^3 \to \mathbf{R}$ satisfies Caratheodory's conditions, $e(x) \in L^1[0,1]$ and g satisfies some additional supplementary conditions. The results of [2] for (1.2), when particularized to the case of (1.1), allow one to consider the possibility that

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