Differential and Integral Equations, Volume 1, Number 1, January 1988, pp. 117–123.

# DIFFERENTIAL EQUATIONS ASSOCIATED WITH COMPACT EVOLUTION GENERATORS

N.H. PAVEL

*Department of Mathematics, University of Texas at Arlington*
*Arlington, Texas 76019, USA*

(Submitted by C. Corduneanu)

**Abstract.** Let $U(t,s)$ be the evolution operator generated by a family of nonlinear, possibly multivalued operators $\{A(t),\ 0 \le t \le T\}$ of dissipative type acting in a Banach space $X$. We prove that if $x \to U(t,s)x,\ s < t \le T$ is compact and $F(t) : \overline{D(A(t))} \to X$ is continuous then the *Cauchy* Problem $u' \in A(t)u + F(t)u,\ u(s) = x_0 \in \overline{D(A(s))}$ has at least an integral solution. One extends the results of Pazy [12] and Vrabie [13] as well as the classical result on the behaviour of the solution as $t \uparrow t_{\max}$.

**1. Statement of main results.** Let $X$ be a real space of norm $\|\cdot\|$. Denote by $J$ the duality mapping of $X$ and set

$$\begin{aligned} \langle y,x\rangle_i &= \inf\{x^*(y);\ x^* \in J(x)\};\ \langle y,x\rangle_- = \langle y,x\rangle_i \|x\|^{-1} \\ \langle y,x\rangle_{\bar{s}} &= \sup\{x^*(y);\ x^* \in J(x)\};\ \langle y,x\rangle_+ = \langle y,x\rangle_{\bar{s}} \|x\|^{-1} \end{aligned} \tag{1.1}$$

where $y,\ x \in X,\ x \neq 0$.

We shall be concerned with the abstract differential equation (inclusion)

$$u'(t) \in A(t)u(t) + F(t)u(t),\ u(s) = x_0 \in \overline{D(A(s))},\ 0 \le t < T \tag{1.2}$$

for some $T > 0$. The hypotheses on $\{A(t)\}$ : For each $t \in [0,T],\ A(t) : D(A(t)) \subset X \to 2^X$ satisfies the range condition

(C1) $R(I - hA(t)) = X$, for all $h > 0$ and $t \in [0,T]$ where $I$ is the identity on $X$.

(C2) There are two continuous functions $f : [0,T] \to X$ and $L : [0,+\infty[\to [0,+\infty[$ such that:

$$\langle y_1 - y_2\,,\ x_1 - x_2\rangle_i \le \|f(t) - f(s)\|\,\|x_1 - x_2\|\, L\Big(\max\{\|x_1\|\,,\ \|x_2\|\}\Big)$$

for all $0 \le s \le t \le T$, $[x_1,\ y_1] \in A(t)$ and $[x_2,\ y_2] \in A(s)$.

(C3) If $t_n \uparrow t$, $x_n \in D(A(t_n))$ and $x_n \to x$, then $x \in \overline{D(A(t))}$ $\big(t_n,\ t \in [0,T]\big)$.

(C4) The evolution operator $U(t,s)$ generated by $\{A(t),\ 0 \le t \le T\}$ is compact (i.e. for every $0 \le s < t \le T$, the operator $x \to U(t,s)x$ maps bounded subsets of $\overline{D(A(s))}$ into relatively compact (precompact) subsets of $X$).

Received August 24, 1987.
*AMS(MOS) Subject Classifications*: 35K22, 34G20.