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DIFFERENTIAL OPERATORS AND THE LEGENDRE TYPE POLYNOMIALS

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Abstract. In 1938, H.L. Krall discovered a fourth-order differential equation that has orthogonal polynomial solutions, called the Legendre type polynomials. Properties of these polynomials and the right-definite problem generated by this fourth-order equation were studied by A.M. Krall in 1981. In this paper, we shall consider this right-definite problem from a different point of view which will enable us to study the fourth-order equation and the polynomials in the left-definite case. As a particular consequence of this study, we shall produce the orthogonality of the derivatives of the Legendre type polynomials. The work in this paper extends earlier work of Titchmarsh, Pleijel and Everitt who studied the right- and left-definite problems associated with the classical Legendre polynomials.

1. Introduction. The second-order differential equations which have the classical orthogonal polynomials as solutions are some of the best examples available to illustrate the well-developed theory of self-adjoint extensions of formally symmetric differential expressions. For a comprehensive study of these second order self-adjoint operators, the reader is referred to the survey paper of Littlejohn and Krall [10] and also to the thesis of Otieno [12]. Besides doing a thorough study of the right-definite boundary value problems associated with the differential equations for these classical orthogonal polynomials, Otieno also considers the left-definite boundary value problems for these equations, extending work of Titchmarsh [15], Pleijel [13] and Everitt [4]. For a detailed analysis of self-adjoint extensions of symmetric operators, the texts of Naimark [11] and Akhiezer and Glazman [1] are recommended. The books of Szegö [14] and Chihara [2] are two excellent sources for properties of orthogonal polynomials in general.

The theory of self-adjoint extensions of formally symmetric differential expressions of order greater than two is considerably more complicated than that of second-order expressions. To illustrate this extension theory with some concrete examples, are there higher order

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