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ON THE ORDER OF MAGNITUDE OF TITCHMARSH–WEYL FUNCTIONS

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Abstract. Upper and lower bounds are obtained for the absolute values of a family of Titchmarsh-Weyl m-coefficients, thereby determining their order of magnitude; only minimal restrictions on the second-order differential operator are imposed. The method also yields the asymptotic behaviour in a certain exceptional case. The results are applied to the estimation of spectral functions.

1. Introduction. Recent progress in the spectral theory of the second-order operator

$$-(py')' + qy = \lambda wy, \quad -\infty < a \leq x < b \leq \infty, \tag{1.1}$$

focussing on the twin concepts of a spectral function and an m-coefficient, has dealt largely with asymptotic approximation to these entities, necessarily with restrictive hypotheses on the coefficients in the differential operator. In the case of the m-coefficient the topic stems from the original order result of Hille [23], improved to an asymptotic formula by Everitt [10]. In one direction these have led the way to higher asymptotics, or indeed asymptotic series for the case $p = w = 1$ (see e.g. [18-20, 25-26]). Another type of development has been to extend the Everitt formula [10] to more general circumstances [1, 2, 11].

The thrust in this paper is in a third direction, going back to the aspect dealt with by Hille. We aim to obtain order-of-magnitude results covering the most general case of (1.1), imposing only the standard requirements for the "right-definite" case. We do not assume any specific asymptotic form for p, q and w as $x \to a$, and do not, in particular require p to be positive. We are concerned with results of the general form

$$C_1\psi(|\lambda|) \leq |m| \leq C_2\psi(|\lambda|) \tag{1.2-3}$$

as $\lambda \to \infty$ in a sector

$$\epsilon \leq \arg \lambda \leq \pi - \epsilon, \tag{1.4}$$

for fixed ϵ with $0 < \epsilon < \pi/2$. In (1.2-3), $\psi(\lambda)$ is a positive function to be specified, actually dependent only on $|\lambda|$.

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