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ON THE INNOVATIONS PROBLEM IN A FINITELY ADDITIVE WHITE NOISE APPROACH TO NONLINEAR FILTERING

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Abstract. Consider the observation of a stochastic signal process in the presence of additive Gaussian white noise in the sense of Balakrishnan. Under the assumptions of signal and noise independence, and $E[\int_0^T \exp(\epsilon|S_t|^2)\,dt] < \infty$ for some $\epsilon > 0$, it is shown that there exists a bijective causal mapping from the observations space to the observations space for the nonlinear filtering problem in a finitely additive white noise framework.

Introduction. It is convenient to begin with a brief review of the nonlinear filtering problem, and the associated problem of the equivalence of the observation and innovation σ – *algebras,* in the Ito framework which is referred to as the classical approach. In this framework the observation noise is modelled as a Wiener process.

Let $S_t(\omega)$ be a stochastic signal process defined on $C[0,1]$ with Wiener measure μ_W defined thereon.

Let $W_t(\omega)$ denote the standard Wiener process on $C[0,1]$ denoting the noise. The observation process is then assumed to be of the form:

$$Y_t(\omega) = \int_0^t S_u(\omega)\,du + W_t(\omega)\,. \tag{1}$$

Let $F_t^Y = \sigma\{Y_s(\omega);\ s \le t\}$ denote the filtration of $Y_.$ up to t or the observation σ – *algebra.*

The innovations problem is to determine whether the innovations process $\nu_t(\omega)$, defined by:

$$\nu_t(\omega) = Y_t(\omega) - \int_0^t \hat{S}_u(\omega)\,du \tag{2}$$

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