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REGULARIZATION OF A STURM–LIOUVILLE PROBLEM WITH AN INTERIOR SINGULARITY USING QUASI-DERIVATIVES

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Abstract. A sturm-Liouville problem with an interior singularity is studied. This problem arose in the study of the eddy motion of the atmosphere about zonally averaged winds. Of the infinitely many self-adjoint operators which can be associated with this problem one is singled out in a natural way. It may have physical significance.

1. Introduction. This paper is essentially a sequel to the earlier paper [5] of Everitt, Gunson and Zettl which in turn arose out of the work of Boyd in [3]. The results here are self contained but reference should be made to [5] for the framework of the problem here considered and for the original contribution made by Boyd and others; in particular, for the physical background of the problem.

We consider the problem of finding solutions of

$$-y''(x) - \frac{1}{x}y(x) = \lambda y(x), \quad x \in [a,b], \quad y(a) = 0 = y(b) \tag{1.1}$$

where the endpoints of the compact interval $[a,b]$ satisfy $-\infty < a < 0 < b < \infty$. The Sturm-Liouville (S–L) eigenvalue program (1.1) has a non-integrable singularity at the point 0, an interior point of the interval $[a,b]$, i.e., $1/|x| \notin L(a,b)$. For this reason the established methods for dealing with regular S–L problems do not apply to (1.1). In particular, solutions y of the equation in (1.1) cannot, in general, be continued through the singular point 0 such that y and y' are continuous there. In fact for some solution y of the differential equation in (1.1) the derivative y' may have a logarithmic singularity at 0, see [5, §4].

In [7] Everitt and Zettl developed a framework in which self-adjoint operators can be associated with S–L problems with interior singularities in a direct sum Hilbert space.

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