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PERIODIC SOLUTIONS OF SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS WHICH APPROXIMATE DELAY EQUATIONS

F. KAPPEL†
Institute for Mathematics, University of Graz, A–8010 Graz, Austria

KLAUS SCHMITT‡
Department of Mathematics, University of Utah, Salt Lake City, Utah 84122 USA

1. Introduction. Let $\alpha > 0$ and $f : \mathbf{R} \to \mathbf{R}$ be given. The scalar delay-differential equation

$$\begin{aligned} \dot{x}(t) &= -\alpha f(x(t-1)), \quad && t \geq 0, \\ x(t) &= \phi(t), && -1 \leq t \leq 0, \end{aligned} \tag{1.1}$$

has been investigated thoroughly since the early sixties and many beautiful results have been found concerning existence of nontrivial periodic solution of this equation (see for instance [8,9,12-14] and the references in these papers). During the past decade one can also see an increasing interest in a class of numerical approximation schemes for delay equations based on abstract approximation results for strongly continuous semigroups of transformations (see for instance [2,3,7]). The various schemes developed during the last years have remarkable qualitative properties. For instance, in case of the scheme developed in [2] and in [7], the approximating ordinary differential systems inherit stabilizability, detectability and exponential stability from the linear delay system which is approximated.

In this paper we study a very simple approximation scheme for (1.1) and demonstrate that the approximating ordinary differential systems inherit from (1.1) the occurence of Hopf-bifurcations and the existence of global branches of nontrivial periodic solution.

The approximation scheme used in this paper was developed in [10]. For $N = 1, 2, \dots$, let $X^N = \{\phi \in C(-1,0;\mathbf{R}) \,|\, \phi$ is a 1st order spline on $[-1,0]$ with knots at the points $-j/N$, $j = 0, \dots, N\}$. Furthermore, let $\pi^N : C(-1,0;\mathbf{R}) \to X^N$ be defined by interpolation at the meshpoints, i.e.

$$(\pi^N \phi)(-\frac{j}{N}) = \phi(-\frac{j}{N}), \quad j = 0, \dots, N,$$

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