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A NOTE ON SEMILINEAR STOCHASTIC EQUATIONS

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Abstract. We study a semi-linear stochastic equation in a Hilbert space with a generalized Wiener process whose covariance is the identity.

0. Introduction. In this paper the following stochastic equation, on a Banach space E,

$$\begin{aligned} dx &= \big(Ax + F(x)\big)\,dt + dW, \\ x(0) &= x \in E \end{aligned} \tag{1}$$

is studied. In equation (1), A stands for the infinitesimal generator of a $C_\circ$-semigroup $S(t)$, $t \geq 0$, F for a locally Lipschitz transformation on E (from a class to be specified later), and W for a "generalized" Wiener process on a Hilbert space $H \supset E$, whose covariance is an identity operator.

Considered problems are of two types. First we obtain existence of a global solution of (1) with regular trajectories. For that purpose we start from the linear equation, corresponding to $F = 0$ and assume that the operator A^{-1} has appropriate radonifying properties. We show that the solution to the linear equation is Hölder continuous as a process with values in a suitable interpolation space. Our Theorem 1 extends regularity results obtained by W.G. Faris and G. Jona-Lasinio [4] and J.B. Walsh [18], for the operator $A = d^2/dx^2$, to general analytic generators. The full nonlinear case is then deduced from a purely deterministic result, Theorem 3, on the following integral equation on E :

$$u(t) = S(t)u_\circ + \int_0^t S(t-s)F(u(s))\,ds + \psi(t), \quad t \geq 0. \tag{2}$$

In (2), ψ is an arbitrary, continuous (or Hölder continuous) function from $[0,\infty)$ into E, such that $\psi(0) = 0$.

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