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OSCILLATION OF EMDEN-FOWLER SYSTEMS*

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Abstract. The oscillation theory of a certain form of systems of two first-order nonlinear differential equations is studied. This form includes in particular the classical Emden-Fowler equations. The well-known oscillation criteria of Atkinson, Belohorec, and Waltman are generalized.

1. Introduction. In the papers [6-8], D.D. Mirzov studies the Emden-Fowler system

$$\begin{aligned} u_1' &= a_1(t)|u_2|^{\lambda_1} \operatorname{sign} u_2 \\ u_2' &= -a_2(t)|u_1|^{\lambda_2} \operatorname{sign} u_1 , \end{aligned} \tag{1.1}$$

with $a_1(t) \geq 0$ or $a_2(t) \geq 0$. A solution is said to be continuable if it exists on the whole half-infinite interval $[0, \infty)$. A continuable solution is said to be oscillatory if it has an infinite number of zeros with ∞ as the only accumulation point. The system (1.1) is said to be oscillatory if every pair of continuable solutions, $u_1(t)$ and $u_2(t)$, are oscillatory.

When $a_1(t) > 0$ and $\lambda_1 = 1$, the system reduces to the classical Emden-Fowler equation:

$$\left(\frac{u_1'}{a_1(t)}\right)' + a_2(t)\,|u_1|^{\lambda_2} \operatorname{sign} u_1 = 0 . \tag{1.2}$$

Mirzov generalizes many of the well-known oscillation criteria for (1.2) to cover (1.1).

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