Differential and Integral Equations, Volume 1, Number 3, July 1988, pp. 377 - 379

ON A SECOND ORDER PERIODIC BOUNDARY VALUE PROBLEM

DANG DINH HAI
Department of Mathematics, Dai Hoc Tong Hop, Ho Chi Minh City University, Vietnam

(Submitted by : Klaus Schmitt)

Abstract. In this note we consider a nonlinear differential equation of second order which has been proposed as a model for the periodic motion of a satellite in its elliptical orbit. We use some elementary methods to demonstrate the existence of an odd periodic solution for a considerably larger parameter set than considered earlier.

Consider the following periodic boundary value problem

$$(1 + a\cos t)x''(t) - 2a\sin t x' + \alpha \sin x = 4a \sin t \tag{1}$$

$$x(0) - x(2\pi) = 0 = x'(0) - x'(2\pi)\,. \tag{2}$$

For the parameter set $\{(a, \alpha) : 0 < a < 1,\ |\alpha| \leq 3\}$, this problem has been proposed in [1] as a mathematical model for the periodic motions of a satellite in the plane of its elliptical orbit. Recently, Petryshin and Yu [4] have studied this problem using the generalized topological degree for A-proper mappings, and have shown the existence of a solution of (1)–(2) for the parameter range

$$0 \leq a < \frac{2}{\pi}|\alpha|, \qquad (8\sqrt{2} + 3)a + 2|\alpha| < 1\,.$$

In a previous paper [2] using variational methods, we showed that in fact, problem (1)–(2) has a solution for $|a| < 1$ and arbitrary α. The aim of this paper is to show that problem (1)–(2) has an odd solution in case $|a| < 1$ and α arbitrary. Since, as will be shown, an odd solution may be obtained by a monotone iteration scheme, our method of proof is in fact constructive. Since we wish to establish the existence of an odd solution, we shall consider equation (1) subject to the boundary condition

$$x(0) = x(\pi) = 0\,. \tag{3}$$

Introducing the new variable

$$y = (1 + a\cos t)x\,,$$

Received November 17, 1987.
Keywords. periodic solutions, odd solutions, supersolution.
AMS(MOS) Subject Classifications: 34B15.