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PERIODIC SOLUTIONS OF PRESCRIBED MINIMAL PERIOD FOR HAMILTONIAN SYSTEMS: AN EXTENSION OF A THEOREM BY EKELAND AND HOFER TO THE NON CONVEX CASE

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0. Introduction. In [4], Ekeland and Hofer proved a very remarkable and general result about the existence of T-periodic solutions having any prescribed minimal period T, for the hamiltonian system

$$\begin{cases} -\dot{x} = \dfrac{\partial}{\partial y} H(x,y) \\ \dot{y} = \dfrac{\partial}{\partial x} H(x,y), \end{cases}$$

in case that H is a convex function on R^{2N} and has a superquadratic behaviour. The proof is essentially based on the idea that the convexity of H, that is the positive definiteness of the quadratic form $\langle H''(z)\varsigma, \varsigma\rangle$, implies a particular relation between the Morse index of a T-periodic solution z_T of (H) and its conjugate points. If z_T is of Mountain Pass type, then this relation enables one to state that T is its minimal period.

In this paper, we assume that H is even and replace the convexity by a weaker assumption of the type

$$\langle H''(z)\varsigma, \varsigma\rangle \geq -c|z|^{\beta-2}|\varsigma|^2 \quad \forall z, \varsigma \in R^{2N}, \ \ \varsigma \neq 0,$$

where c is a positive constant number and $\beta > 2$ is the superquadraticity exponent of H. Roughly speaking, it can be seen that this assumption still suffices to give the same kind of behaviour for the Morse index of a periodic solution and still yields the same existence result of T-periodic solutions having any prescribed minimal period T, provided that the coefficient c is chosen small enough with respect to the growth coefficients of H. Actually, the problem is solved, as in [4], not in a direct way, but through the use of a suitable version of the well known dual action principle by Clarke and Ekeland (see [3]). An essential tool for the proof of our result is a theorem proved in [6], about the existence of solutions of prescribed minimal period for a Hamiltonian system related to a Hamiltonian function which is the sum of a term of the type H (as in [4]) plus a quadratic term. Finally, we wish to point out that in the present paper we are essentially interested in the extension of the result given

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