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A NONLINEAR EQUATION WITH PIECEWISE CONTINUOUS ARGUMENT

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Abstract. Asymptotic and qualitative behavior of solutions is established for the equations (1) $x'(t) = \mu x(t)(1-x([t]))$, (2) $x'(t) = \mu x(t)\big(1-x\big(2[(t+1)/2]\big)\big)$, where μ is a positive parameter. Comparison is made with the continuous logistic equation (3) $x'(t) = \mu x(t)(1-x(t))$ and the discrete logistic equation (4) $x_n = \mu x_{n-1}(1-x_{n-1})$. One result is that (1) and (4) can exhibit complicated dynamics and (2) and (3) cannot.

I. Introduction. This paper is devoted to a study of two scalar non-linear differential equations of the logistic form, in which one of the arguments is t and the other argument is a piece-wise continuous function of t. Specifically, the equations are as follows:

$$x'(t) = \mu x(t)(1-x([t])), \quad x(0) = c_0, \quad t \geq 0, \tag{1.1}$$

$$x'(t) = \mu x(t)(1-x(2[(t+1)/2])), \quad x(0) = c_0, \quad t \geq 0, \tag{1.2}$$

Here, x' is the derivative of x, $[t]$ denotes the greatest integer function, $[t] = n$ when $n \leq t < n+1$ where n is an integer, and μ and c_0 are real parameters.

Equations with arguments less than t, such as $[t]$ in (1.1), may be regarded as special types of functional differential equations with retarded argument. These have been studied in some linear and nonlinear cases by Cooke and Wiener [2,3]. Equations of the neutral type with this kind of argument have also been discussed by these authors in [5], and equations of the advanced type were investigated by Shah and Wiener [8]. Cooke and Wiener also introduced an example of an equation with argument $2[(t+1)/2]$, which is alternately advanced and retarded [4].

It is possible to think of (1.1) and (1.2) as semi-discretizations of

$$x'(t) = \mu x(t)(1-x(t)), \quad x(0) = c_0, \quad t \geq 0, \tag{1.3}$$

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