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SOME DISCONTINUOUS VARIATIONAL PROBLEMS

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0. Introduction. Differential equations with discontinuous nonlinearities have been investigated by several authors. We cite [2, 10, 11, 18, 21, 22] and, for specific models in mathematical physics, [4, 5, 9, 12, 16]. In this paper, we study elliptic boundary value problems of the type:

$$\begin{aligned} -\Delta u &= h(u-a)p(u) && \text{in } \Omega \\ u &= 0 && \text{on } \partial\Omega, \end{aligned}$$

where h is the Heaviside function, $a > 0$, $p \geq 0$ is nondecreasing in u and Ω is a smooth domain in $\mathbf{R}^n$. In applications, p may depend on $x \in \Omega$ and on parameters as well. We focus on nonlinearities for which, besides the zero solution, there are two additional solutions corresponding to a minimum and to a saddle point found using the "Mountain Pass Theorem" [3].

As in [2], the approach we use is that of a dual variational principle ([13], see also [14]), which yields a functional which is more regular than that arising from a direct variational principle involving a Dirichlet integral. The dual principle is briefly discussed in §1 and existence results are proven in §2. In §3 it is shown that if the domain Ω has some symmetry and if a compatible symmetry exists in the nonlinear term, then in this general setting, the solutions inherit the symmetry. The simplicity of the dual approach is especially evident in this section.

From among the possible applications, we have chosen to discuss in §4 a problem arising in plasma physics. It is a model for the distribution of temperature in an electric arc for which the constitutive law contains a discontinuity (cf. [19], [12]). In contrast with [12], we work with an unconstrained problem and can freely vary parameters. Here a represents the difference between the temperature of a containing cylinder and that of discharge. It is

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