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CONCAVITY AND MONOTONICITY PROPERTIES OF SOLUTIONS OF EMDEN-FOWLER EQUATIONS*

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Abstract. This article is concerned with concavity and monotonicity properties of solutions of boundary value problems of the type

$$u'' = p(x)u' + f(u), \quad x > 0; \quad u(0) = \gamma, \; u'(0) = 0.$$

The coefficient p may be singular near the origin. It is assumed that there exists a u_0 such that $f(u) < 0$ if $u > u_0$ and $f(u_0) = 0$. Particular cases are $f(u) = 1 - e^u$; $f(u) = u/(q-1) - |u|^q \operatorname{sign}(u)$, where $1 < q < \infty$; and $f(u) = -u/(1-q) + |u|^q \operatorname{sign}(u)$, where $0 < q < 1$. The results generalize and extend those of Friedman, Friedman, and McLeod [*Concavity of solutions of nonlinear ordinary differential equations,* preprint 1987].

1. Introduction. In this article we are interested in concavity properties of solutions u of boundary value problems of the following type:

$$u'' = p(x)u' + f(u), \quad x > 0; \quad u(0) = \gamma, \; u'(0) = 0. \tag{1}$$

The coefficient p may be singular near the origin. The (nonlinear) function f is defined on $(-\infty, \infty)$ and there exists a u_0 such that

$$f(u) < 0 \text{ if } u > u_0, \quad f(u_0) = 0. \tag{2}$$

We assume throughout that $p \in C^2((0,\infty))$ and $f \in C^2((-\infty,\infty))$; other conditions are imposed as needed. The differential equation in (1) belongs to the class of Emden-Fowler equations.

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