

## ON LIOUVILLE GREEN (WKB) APPROXIMATION FOR SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS

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**Abstract.** Such an approximation is obtained without hypotheses on  $f''(x)$  and without using the general linear system approach.

**1. Introduction.** Let  $f(x)$  be positive and continuously differentiable on the positive halfaxis. It is known, as a result of extensive research, that (positive) solutions  $y(x)$  of the equation

$$y'' = f(x)y \tag{1}$$

can be approximated by the functions (called WKB solutions, or Liouville-Green approximation, [4])

$$\phi_i(x) = f^{-1/4}(x) \exp \left\{ \pm \int_a^x f^{1/2}(t) dt \right\}, \quad i = 1, 2,$$

( $a > 0$ , arbitrary, fixed) in the sense that, for  $x \rightarrow \infty$ ,  $y(x) \sim \phi(x)$  (i.e.,  $y/\phi \rightarrow 1$ ). This is, as a rule, proved by imposing some hypotheses on  $f''(x)$  (cf.e.g. [1, 3, 6]). We quote, among these results the following, as a typical and the most general one:

**Theorem A.** ([1, Th. IV, 14]). *Let  $f(x)$  be a positive, twice continuously differentiable function for  $x \geq a$  and such that*

$$\int_a^\infty |f^{-3/2} f''| dx < \infty, \tag{2}$$

*then, (1) has a fundamental system of solutions satisfying for  $x \rightarrow \infty$ ,  $i = 1, 2$ ,*

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