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HIGHLY DEGENERATE PARABOLIC BOUNDARY VALUE PROBLEMS*

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Abstract. Of concern are parabolic equations of the form

$$\partial u/\partial t = \phi(x, \nabla u)\Delta u \qquad (x \in \Omega \Subset \mathbb{R}^n,\ t \geq 0)$$

where $\phi(x, \xi) > 0$ on $\Omega \times \mathbb{R}^n$ but $\phi(x, \xi) \to 0$ very rapidly as $x \to \partial\Omega$. By associating the Wentzel boundary condition with this equation, the initial value problem is shown to be well-posed. This is done with the aid of the Crandall-Liggett theorem, applied in the space $C(\overline{\Omega})$.

1. Introduction. In the 1950s, W. Feller [9-11], working from a semigroup point of view, determined all one dimensional Markov processes of diffusion type. If $[a, b]$ denote the underlying spatial interval, Feller classified the boundary points (a and b) as being of regular, exit, entrance, or natural type. (For a nice introduction to these ideas from an analyst's point of view see Yosida's book [27].) Among the attractive interpretations was that a diffusing particle could not reach an entrance boundary in finite time, and consequently no boundary condition need be imposed at such a point. The kind of boundary conditions associated with the (weakly) elliptic generators were sometimes of a nonlocal character, and Feller termed them *lateral conditions* rather than boundary conditions. Soon afterwards, A.D. Wentzel [26] began a program of finding boundary and lateral conditions which characterize multidimensional diffusion. A feature of his work was the use of *second* derivatives in the boundary conditions. Later contributions to this theory were made by Sato and Ueno [21], Taira [22-23], and many others. An earlier contribution from a nonprobabilistic point of view was made by Vishik [24].

A clean semigroup version of these results in one space dimension was obtained recently by Ph. Clément and C.A. Timmermanns [6]. Building upon earlier work of Martini and Boer [17-19], they established the following result.

Let α, β be continuous real functions on the open interval $(0,1)$ with α positive. Define a linear operator A on the real Banach space $X = C[0,1]$ by

$$Au = \alpha u'' + \beta u'$$

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