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A SPECTRAL MAPPING THEOREM FOR POLYNOMIAL OPERATOR MATRICES

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Abstract. Systems of linear evolution equations can be written as a single equation

$$\dot{u}(t) = \mathcal{A}u(t), \tag{*}$$

where u is a function with values in a product space E^n and $\mathcal{A} = (A_{ij})_{n\times n}$ is an operator matrix. Often the entries A_{ij} are polynomials $p_{ij}(A)$ with respect to a single (unbounded) operator A on E (see, e.g., [1], [2], [3], [6], [11]). In order to solve (*) one has to determine the properties of the operator matrix $\mathcal{A}$. In particular one has to find an appropriate domain $D(\mathcal{A})$ such that $\mathcal{A}$ is closed. This will be discussed in the first part of this paper. Then it is important to compute the spectrum $\sigma(\mathcal{A})$ of $\mathcal{A}$. One expects a kind of spectral mapping theorem based on the spectrum $\sigma(A)$ of A and the structure of the matrix (p_{ij}). We show in Part 2 in which sense such a spectral mapping theorem holds. An application to stability theory, i.e., the computation of an estimate for the spectral bound $\mathrm{s}(\mathcal{A})$ concludes this paper. In a subsequent paper we discuss which operator matrices $(p_{ij}(A))$ generate strongly continuous semigroups on E^n and give applications to systems of differential equations.

1. How to define an operator matrix? We study $n \times n$ matrices $\mathcal{A}$ whose entries are polynomials $p_{ij}(A)$ in a fixed – possibly unbounded – operator A on some Banach space E.

For bounded A it is obvious that the operator matrix $\mathcal{A}$ defines a bounded operator on the product space $\mathcal{E} := E^n$. The situation for unbounded A is more complicated. In fact, the matrix $\mathcal{A}$ only induces a formal map

$$x = (x_1, \cdots, x_n)^t \mapsto \mathcal{A}x = \left(\sum_{i=1}^n p_{1i}(A)x_i, \dots, \sum_{i=1}^n p_{ni}(A)x_i\right)^t,$$

but leaves open a wide choice of possible domains. If one wants "nice" properties of $\mathcal{A}$, such as closedness for example, then a more careful analysis is necessary.

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