Differential and Integral Equations, Volume 2, Number 2, April 1989, pp. 193-202.

MULTIPLICITY OF kT-PERIODIC SOLUTIONS NEAR A GIVEN T-PERIODIC SOLUTION FOR NONLINEAR HAMILTONIAN SYSTEMS*

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1. Introduction and results. In this note we consider the question of existence and multiplicity of periodic solutions with minimal period kT ($k \in \mathbb{N}$) near an equilibrium ($z \equiv 0$) for systems of the form

$$\dot{z} = J\nabla H(t, z), \quad z \in \mathbb{R}^{2n}. \tag{1.1}$$

Here J is the skewsymmetric matrix

$$J = \begin{pmatrix} 0 & Id \\ -Id & 0 \end{pmatrix} \in L(\mathbb{R}^{2n}),$$

∇ stands for the gradient in the z-variable, $H \in C^2(\mathbb{R} \times \Omega, \mathbb{R})$ (with Ω a neighborhood of $0 \in \mathbb{R}^{2n}$) is the Hamiltonian function, which is assumed to depend periodically on time

$$H(t+T, z) = H(t, z) \quad T > 0 \ \forall t \in \mathbb{R}, \ \forall z \in \mathbb{R}^{2n},$$
$$H(t, 0) \equiv 0 \qquad \forall t \in \mathbb{R}$$

and

$$\nabla H(t, 0) \equiv 0 \qquad \forall t \in \mathbb{R}.$$

Such a problem occures, for example, if we know a T-periodic solution $\bar{x}$ of an Hamiltonian system whose Hamiltonian does not depend explicitly on time and if we look for periodic solutions nearby, having period kT.

The existence of these solutions requires assumptions on the linearized part of the equation at 0 and also on the nonlinear part. Let the Taylor expansion of the function H at 0 be

$$H(t, z) = \frac{1}{2} \langle Qz, z \rangle + \hat{H}(t, z) \tag{1.2}$$

where $\hat{H}(t, z) = o(|z|^2)$ uniformly in t. We shall assume for the linearized system

$$\dot{z} = JQz$$

Received June 12, 1988.
*Partially supported by G.N.F.M. - C.N.R.
AMS Subject Classifications: 58F22, 58F05, 34C25.