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QUALITATIVE SIMULATION OF DIFFERENTIAL EQUATIONS

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Abstract. This paper deals with the QSIM algorithm introduced by Kuipers (see [9])to track down the changes of monotonicity properties of solutions to a differential equation or other observations of the solutions. It introduces the concept of "qualitative cells" where the monotonicity properties of the states of the system remain the same. It provides sufficient conditions for the non emptiness of such cells, for their singularities, for the transition from one cell to another, and characterizes also the qualitative equilibria and repellors of the associated qualitative system.

Introduction. The purpose of this paper is to revisit the QSIM algorithm introduced by Kuipers in [9] for studying the qualitative evolution of solutions to a differential equation

$$x'(t) = f(x(t)) \tag{1}$$

where the state x ranges over a closed subset K of a finite dimensional vector-space $X := \mathbb{R}^n$.

We recall that the *contingent cone* $T_K(x)$ to a subset K at $x \in K$ is the closed cone of elements v satisfying

$$\liminf_{h \to 0+} \frac{d(x+hv, K)}{h} = 0$$

and that K is a viability domain if and only if

$$\forall\, x \in K, \quad f(x) \in T_K(x).$$

We posit the assumptions of Nagumo Theorem:

Theorem. [Nagumo]. Under assumptions

$$\begin{cases} i) & F \text{is continuous with linear growth} \\ ii) & K \text{is a closed viability domain,} \end{cases} \tag{2}$$

a closed subset K is a viability domain if and only if it enjoys the viability property: for any initial state $x_0 \in K$, there exists a solution $x(\cdot)$ to the differential equation (1), which is *viable* in the sense that $x(t)$ remains in K for all $t \geq 0$. [1, Chapter 4].

The *qualitative state* of a solution to the differential equation (1) at a given time t is the knowledge of the monotonicity property of each component $x_i(t)$ of a solution $x(\cdot)$ to

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