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AN EXACT FORMULA FOR THE BRANCH OF PERIOD-4-SOLUTIONS OF $\dot{x} = -\lambda f(x(t-1))$ WHICH BIFURCATES AT $\lambda = \pi/2$

O. ARINO AND A.A. CHÉRIF

Département de Mathématiques, Faculté des Sciences, 64000 Pau, France and Department of Mathematics, University of Mississippi, University, MS 38677 USA

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Introduction. In this paper, we construct an exact formula for the periods of solutions of the following system of ordinary differential equations

$$\begin{cases} \dot{x} = -\lambda f(y(t)) \\ \dot{y} = \lambda f(x(t)). \end{cases} \tag{1}$$

Throughout the paper, we assume that f is a function which satisfies the following conditions: $f(x)x > 0$ for $x \neq 0$, f is odd and differentiable, $f'(0) = 1$. The formula we obtain for the periods is a function $T(x_0, \lambda)$ of λ and $x_0 > 0$ where $(x_0, 0)$ is any initial data for system (1). It is known (see [4]) that period-4-solutions of (1) yield period-4-solutions of

$$\dot{x} = -\lambda f(x(t-1)). \tag{2}$$

For these solutions, $T(x_0, \lambda)$ yields a function $\lambda(x_0)$ which shows that a branch of period-4-solutions of (2) bifurcates at $\lambda = \pi/2$. Using $\lambda(x_0)$, we show that the higher derivatives of f at 0 determine the behaviour of the branch near $\pi/2$.

Section 1. In this section, we construct the formula for periods of the solutions of system (1) with initial value of the form $(x(0), y(0)) = (x_0, 0)$, where $x_0 > 0$. We assume that there exists a real interval I containing 0 and that f verifies the following hypothesis on I :

$$\{f \text{ is odd and } f \in C^1(I),\ f(x)x > 0 \text{ for } x \neq 0, \text{ and } f'(0) = 1\}. \tag{H1}$$

Proposition 1.1. *Assume that f satisfies the hypothesis (H1) and consider the following initial value problem*

$$\begin{cases} \dfrac{dx}{dt} = -\lambda f(y(t)) \\ \dfrac{dy}{dt} = \lambda f(x(t)) \\ (x(0), y(0)) = (x_0, 0) \quad \text{where } x_0 \in I. \end{cases} \tag{1}$$

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