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STABILIZING SECOND ORDER DIFFERENTIAL EQUATIONS

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Abstract. This paper concerns the stabilization of a second differential controlled equation in R^n, $x'' + \partial\phi(x) \ni u$, $\|u(t)\| \leq 1$ by a feedback law of the form $u = \psi(x')$. Applications to the stabilizations of the movement of an elastic string with a discrete distribution of masses and limited from below by a rigid obstacle is given.

1. Introduction. This work is concerned with the controlled second order differential equation

$$\frac{d^2x}{dt^2} + \partial\phi(x) \ni u \quad \text{in } R^+ \tag{1.1}$$

where $x : [0,T] \to R^n$, $\|u(t)\| \leq 1$ a.e., $t > 0$, $x'' = d^2x/dt^2$ and $\partial\phi : R^n \to 2^{R^n}$ is the subdifferential of a lower semicontinuous convex function $\phi : R^n \to (-\infty, +\infty]$; i.e., (see e.g. [2])

$$\partial\phi(x) = \{y \in R^n;\ \phi(x) - \phi(u) \leq \langle y, x - u\rangle \ \ \forall u \in R^n\}\,.$$

We have denoted by $\langle\cdot,\cdot\rangle$ the usual scalar product in R^n and by $\|\cdot\|$ the Euclidean norm of R^n.

The main result, Theorem 2 below, amounts to saying that the above equation can be stabilized by a nonlinear feedback law $u = \psi(dx/dt)$.

By solution to the Cauchy problem

$$\frac{d^2x}{dt^2} + \partial\phi(x) \ni \psi(\frac{dx}{dt}) \quad \text{in } (0,T) \tag{1.2}$$

$$x(0) = x_0,\ \ \frac{dx}{dt}(0) = x,$$

we mean a function $x \in W^{1,\infty}([0,T]; R^n)$ such that (see [3])

(a) $d^2x/dt^2 + \mu - \psi(dx/dt) = 0$ in the sense of distribution where μ is a bounded measure on $[0,T]$ such that

$$\int_0^T (\phi(v(s)) - \phi(x(s)))\, ds \geq \mu(v - x) \tag{1.3}$$

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