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ON THE EXISTENCE OF A MAXIMAL WEAK SOLUTION FOR A SEMILINEAR ELLIPTIC EQUATION

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0. Introduction. Consider the semilinear problem

$$\begin{cases} -\Delta u = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{0.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ and $f : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$. It is well known, see e.g. [16], that for $f \in C^1(\overline{\Omega} \times \mathbb{R})$ and hence solutions in $C^{2+\vartheta}(\Omega)$, there is a solution in between a sub- and a supersolution. (The supersolution has to lie above the subsolution) There the super- and subsolutions are assumed to be in $C^2(\Omega)$. Similar results for sub- and supersolutions in $W^{2,p}(\Omega)$ are shown in [5, 6].

A first place where a weaker supersolution is used is [13]. Deuel and Hess established existence of a solution between weaker sub- and supersolutions in [10].

Amann showed in [3, 4] for the classical case ($u \in C^2(\Omega) \cap C^{\vartheta}(\overline{\Omega})$) in fact the existence of a minimal and a maximal solution between a sub- and a supersolution in $C^{2+\vartheta}(\overline{\Omega})$.

The classical proofs can be extended to functions f which are Lipschitz. In this note we will show that the result is still true even if f is not Lipschitz. In section 1 we will use super (sub) solutions in $C(\overline{\Omega})$. In section 2 we will use super (sub) solutions in $W^{1,2}(\Omega)$ and allow general bounded domains. Neither definition of super (sub) solution is included in the other even for regular domains, though a $C_0(\overline{\Omega})$-solution is necessarily a $W_0^{1,2}(\Omega)$-solution. Thus neither of our two main results is included in the other.

1. A maximal solution in $C(\overline{\Omega})$. In this section we consider (0.1) for functions f in $C(\overline{\Omega} \times \mathbb{R})$. Moreover we assume that every boundary point is regular with respect to the Laplacian. A boundary point is regular if there exists a barrier-function at that point. For a definition see [12, p. 25]. In this section we are interested in solutions u in $C(\overline{\Omega})$.

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