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WIENER ESTIMATES FOR DEGENERATE ELLIPTIC EQUATIONS II

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Abstract. We prove a Wiener estimate for the modulus of continuity at a boundary point for a solution of a Dirichlet problem relative to a degenerate elliptic operator in divergence form, which is coercive with respect to a weight in the A_2 Muckenhoupt's class.

1. Introduction. The purpose of this paper is to prove an estimate on the modulus of continuity at the boundary points for weak solutions of the Dirichlet problem (i.e., a Wiener estimate) relative to the following elliptic equation

$$Lu = -D_j(a_{ij}(x)D_i u) = 0 \tag{1.1}$$

($D_i = D_{x_i}$ and the convention on summation of repeated indices is used) where Ω is a bounded open set of $\mathbb{R}^N$, $N \geq 3$, and $a_{ij}(x)$, $i,j = 1,\cdots,N$, is a symmetric matrix of measurable functions on $\mathbb{R}^N$ such that

$$\lambda w(x)|\xi|^2 \leq a_{ij}(x)\xi_i\xi_j \leq \Lambda w(x)|\xi|^2. \tag{1.2}$$

We assume that the function w in (1.2) is a nonnegative weight in the A_2 Muckenhoupt's class (for the definition of this class of weight, see [2]). The authors have been concerned with this problem in a previous paper, where an estimate on the modulus of continuity at the boundary point x_0 for weak solutions of (1.1) has been obtained under the additional assumption

$$w(B(r,x_0))(w(B(R,x_0)))^{-1} \leq (r/R)^{2+\epsilon} \tag{1.3}$$

($w(E) = \int_E w(x)\,dx$ and $B(r,x)$ is the ball of center x and radius r), $0 \leq r \leq R$, $\epsilon > 0$, [1]; here we will avoid the assumption (1.3).

We observe that (1.3) does not allow a positive capacity for the point x_0; our difficulties come here from the existence of points of this type, which prevents us from estimating the behavior of a solution in a ball by its behavior in the anulus.

Finally, we recall that a qualitative Wiener criterion for weak solutions of (1.1) has been proved in [3] by E. Fabes, D. Jerison, C. Kenig.

In the following $L^2(\Omega; w)$, $H^{1,p}(\Omega, w)$, $H^1(\Omega, w)$, $H^1_0(\Omega, w)$ are the usual weighted spaces L^2, $H^{1,p}$, H^1, H^1_0. We recall now the notion of capacity related to the weighted H^1_0 space:

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