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BOUNDARY VALUE PROBLEMS FOR SECOND ORDER DIFFERENTIAL EQUATIONS AND A CONVEX PROBLEM OF BOLZA

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Abstract. In this paper we are concerned with three types of problems. (I) Existence and uniqueness of the solution u of the following boundary value problems:

$$p(t)u''(t) + r(t)u'(t) \in Au(t) + f(t), \quad \text{a.e. on } [0,T],\ T > 0 \tag{1}$$

$$u'(0) \in \alpha(u(0) - a), \ \ u'(T) \in \beta(u(T) - b) \tag{2}$$

$$u''(t) \in Au(t) + f(t), \quad \text{a.e. on } [0,T] \tag{3}$$

$$u(0) = u(T), \ \ u'(0) - u'(T) \in \gamma(u(0)) \tag{4}$$

$$u''(t) \in Au(t) + f(t), \quad \text{a.e. on } [0,T] \tag{3}$$

$$u'(0) = u'(T), \ \ u(0) - u(T) \in \delta(u'(0)). \tag{5}$$

Here, A, α, $-\beta$, γ, δ are maximal monotone (possibly multivalued) operators acting in a real Hilbert space H, $a, b \in D(A)$, $T > 0$ arbitrary, $f \in L^2([0,T]; H)$ $\big(L^2$-with the weight function $\tilde{r}/p$, where $\tilde{r}(t) = \exp\big(\int_0^t (r(s)/p(s))\, ds\big)$, p, $r : [0,T] \to \mathbb{R}$ continuous with $p(t) \geq c > 0\ \forall\, t \in [0,T]$.

(II) Continuous dependence of $u = u(t,a,b,f)$ on a, b and f.

(III) In the case in which A, α and $-\beta$ are subdifferentials of some lower semi-continuous convex (l.s.c.) proper functions, we prove the equivalence of (1)-(2) with a convex problem of Bolza (Theorem 3.3).

1. Introduction. This paper contains several results on existence, uniqueness, continuous dependence on initial data and some optimization problems and application to some elliptic equations. The idea to work in $\mathcal{L}^2_{\tilde{r}/p} = L^2$ with the weight function $\tilde{r}$ enables one to eliminate the differentiability assumption on p and r in Theorem 3.1, and to prove the equivalance of (1)-(2) with an optimization problem. In section 2, we present some preliminary results which help to carry out the proof of the main results. Some of these results (e.g. Lemma 2.1 and Proposition 2.1) seem to be new. Theorem 3.1 in section 3 contains Theorem 3.1 of Veron [11]. There, he assumes $p \in W^{2,\infty}[0,T]$, $r \in W^{1,\infty}[0,T]$, while in this paper we assume only the continuity of p and r and we obtain the same conclusion. The result given in Theorem 3.3 is different from those of Barbu [4, p. 301], see Remark 3.4. Boundary conditions of the form (2) have been considered by Brezis [5] in the case $\alpha = \partial j_1$

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