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REGULARITY PROPERTIES OF SOLUTIONS TO HAMILTON-JACOBI EQUATIONS IN INFINITE DIMENSIONS AND NONLINEAR OPTIMAL CONTROL

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Abstract. This paper is concerned with the semi-concavity properties of the value function $V(t,x)$ of an Optimal Control Problem (in Bolza form) for a Distributed Parameter System governed by the semilinear State Equation

$$\begin{aligned} y'(s) &= Ay(s) + F\big(y(s)\big) + Bu(s), \quad t \le s \le T \\ y(t) &= x \in X \quad u : [t,T] \to U. \end{aligned} \tag{SE}$$

Here, both the State Space X and the Control Space U are Banach spaces, A is the infinitesimal generator of an analytic semigroup on X and F is a nonlinear perturbation, possibly defined on a dense subspace of X.

By using regularity results for solutions to (SE), we obtain one-sided bounds on V of the form

$$\lambda V(t,x_1) + (1-\lambda)V(t,x_0) - V\big(t,\lambda x_1 + (1-\lambda)x_0\big) \le C\lambda(1-\lambda)|x_1 - x_0|^2 \tag{SC}$$

for all $\lambda \in [0,1]$.

The above estimate is also applied to analyze the structure of the generalized gradient $\partial_x V(t,x)$ and to derive the Feedback Formula.

1. Introduction. Let X be a separable reflexive Banach space with norm $|\ |$, which is assumed to be continuously differentiable in $X\backslash\{0\}$. We denote by X^* the dual of X, the duality pairing being represented by $\langle , \rangle$.

We are interested in the Hamilton-Jacobi equation

$$-\frac{\partial V}{\partial t}(t,x) + H\big(B^*\nabla V(t,x)\big) - \langle Ax + F(x), \nabla V(t,x)\rangle - g(x) = 0 \tag{1.1}$$

in $[0,T] \times X$, with terminal data

$$V(T,x) = \phi(x).$$

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