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PARABOLIC INTEGRODIFFERENTIAL EQUATIONS WITH NONHOMOGENEOUS BOUNDARY CONDITIONS

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Abstract. We consider a parabolic partial integrodifferential Volterra equation with nonhomogeneous boundary conditions

$$\begin{cases} u_t(t,x) = \Delta u(t,x) + \int_0^t k(t-s)\Delta u(s,x)\,ds + f(t,x), & t \in [0,T],\ x \in \Omega \\ u(0,x) = u_0(x), & x \in \Omega \\ u(t,x) = \varphi(t,x), & t \in [0,T],\ x \in \partial\Omega \end{cases} \tag{*}$$

and a similar problem with infinite delay, where Δ is the Laplace operator and $k : [0,+\infty[\to \mathbb{R}$. Under suitable assumptions on the kernel k, we state some results about the existence, uniqueness and regularity of the solutions of (*) and of the equation with infinite delay.

0. Introduction. This paper deals with a class of parabolic partial integrodifferential Volterra equations with nonhomogeneous boundary conditions

$$\begin{cases} u_t(t,x) = \Delta u(t,x) + \displaystyle\int_0^t k(t-s)\Delta u(s,x)\,ds + f(t,x), & t \in [0,T],\ x \in \Omega \\ u(0,x) = u_0(x), & x \in \Omega \\ u(t,x) = \varphi(t,x), & t \in [0,T],\ x \in \partial\Omega \end{cases} \tag{0.1}$$

and the similar problem with infinite delay

$$\begin{cases} u_t(t,x) = \Delta u(t,x) + \displaystyle\int_{-\infty}^t k(t-s)\Delta u(s,x)\,ds + f(t,x), & t \in \mathbb{R},\ x \in \Omega \\ u(t,x) = \varphi(t,x), & t \in \mathbb{R},\ x \in \partial\Omega \end{cases} \tag{0.2}$$

where $T > 0$, Ω is a bounded open set in $\mathbb{R}^n$, $n \in \mathbb{N}$, with regular boundary $\partial\Omega$, Δ is the Laplace operator and the Laplace transform of $k : [0,+\infty[\to \mathbb{R}$ verifies suitable assumptions.

Many authors have studied problems (0.1) and (0.2) when $\varphi \equiv 0$ using Laplace transform and other methods; see for instance [1]-[5], [8]-[10].

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