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EXISTENCE OF STEADY FLOWS OF VISCOELASTIC FLUIDS OF JEFFREYS TYPE WITH TRACTION BOUNDARY CONDITIONS

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Abstract. We consider steady flow of a viscoelastic fluid transverse to a strip. The fluid has a differential constitutive relation of Jeffreys type. We prescribe the tractions on both boundaries of the strip as well as the extra stresses at the inflow boundary. Existence of steady flows is proved under the assumption that the data are small and inertial effects are neglected.

1. Introduction. While the existence theory of steady solutions of the Navier-Stokes equations is well established (see [5]), very little is known about steady flows of viscoelastic fluids. Even the existence of flows perturbing a state of rest or a uniform flow has been investigated only recently [2]-[4]. There are basically two features which make this problem difficult:

1. The problem involves a singular perturbation. If, for example, we linearize at the rest state, then we obtain the Stokes problem. However, the nonlinearities introduce terms of higher differential order than those which are present in the linear problem. Therefore, we cannot apply the implicit function theorem to obtain an existence theorem for small data.
2. Viscoelastic fluids have memory, and the flow in the domain under consideration depends on the deformations experienced by the fluid before it entered the domain. This makes it necessary to prescribe boundary conditions at an inflow boundary in addition to those which are required in the Newtonian case. For a general fluid with memory an infinite number of such inflow boundary conditions would be required. However, for fluids with differential constitutive relations of Maxwell or Jeffreys type the necessary information about the flow history is contained in the extra stresses; i.e., in a finite number of variables, and a boundary value problem with a finite number of boundary conditions can be formulated.

Maxwell models are characterized by an ordinary differential equation along streamlines which relates the extra stress to the velocity gradient; Jeffreys models have a Newtonian contribution to the extra stress plus an additional term which is like the stress for a Maxwell model. Differential models are popular for numerical simulations. Here the question of inflow

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